

IV. Mathematical expressions

Since correct typographical presentation is crucial to understanding equations, authors of mathematical papers should read this section of the manual with special care: its aim is to inform authors of the limits of typesetting resources and to indicate standard methods of presentation that will ensure correct and efficient typesetting.

A. AVAILABILITY OF CHARACTERS FOR TYPESETTING

(1) The latin and greek alphabets available for use in technical material are described in Appendix E. The latin alphabet is available in five type styles, or "fonts": roman (upright), italic (oblique or slanted to the right), script (cursive), german (Fraktur), and sans serif (unadorned). Examples of standard lightface type and boldface (heavy) type are given in Appendix E (see also Sec. IV B 3).

(2) Appendix F is a catalog of available special symbols, grouped and numbered according to type: diacritics (accents), parentheses and brackets, mathematical signs of relation, mathematical operators, phonetic symbols, figure symbols, etc. Any of these symbols may be used if necessary, but it must be emphasized that, with due respect for accepted practice, mathematical notation should be kept as simple as possible.

(3) Any diacritical sign in Appendix F can be placed over (or under, or through) any alphanumeric character, including subscripts and superscripts:

$$\bar{B}, \underline{2}, \hat{\lambda}, \underline{T}''$$

Two levels of diacritical signs are possible, as in

$$\hat{\underline{a}}, \underline{\hat{y}}.$$

Three levels are to be avoided.

(4) Symbols not listed in Appendix F can sometimes be specially made, but the cost is high and delay in publication will result. Use an alternative symbolism if possible.

(5) A mathematical expression used repeatedly throughout a paper should be defined in terms of an appropriate symbol the first time it appears. Then the symbol can be used as shorthand for the expression. Look for opportunities to use this technique for shortening the paper and reducing the chance of errors in typesetting.

B. MARKING OF MATHEMATICAL MATERIAL

The importance of legible mathematical material cannot be overemphasized. Keep in mind that typesetters will reproduce what they see, not what you know. Since typed or printed material is far more legible than hand writing, type or print as much of the mathematics as possible. Handwritten material must be neatly lettered in black ink.

1. Clarification of characters

(1) Many handwritten letters, numbers, and special symbols look alike and are hard to identify. Especially troublesome cases are given in Table V. Each should be identified the first time it occurs in manuscript, and thereafter if any ambiguity is still possible. Write the identification in black

pencil above the symbol or in the left margin. Special symbols are most efficiently identified by using the identification numbers given in Appendix F:

$$E \overset{R11}{\leq} 36 \text{ MeV}$$

Always identify handwritten script, german, and sans serif letters the first time they appear in manuscript.

(2) Typed or printed material has the advantages of legibility and consistent formation of characters, but some confusion is still possible. On many typewriters the letter l and the number 1 are the same, and the author must distinguish between them: loop the letter l by hand when it is used as a mathematical symbol and, as a safeguard, identify both symbols (as "lc ell" and "one") the first time they appear. Similar precautions should be taken to distinguish between a capital O and a zero and between a capital X and a multiplication sign.

2. Roman versus italic type

(1) According to conventional practice, latin letters used as mathematical symbols are set in italic type to distinguish them from ordinary roman text. The typesetter is therefore instructed to set in italic type all unmarked latin letters that are obviously not words:

$$\text{For } 2x_r e^x = 0, \text{ we obtain}$$

will automatically be set as

$$\text{For } 2x_r e^x = 0, \text{ we obtain}$$

Authors should mark a letter for italic type (by underscoring in pencil) only if the letter might be mistaken for a word:

When a is a number dependent on...

(2) Since words will automatically be set in roman type, authors should mark words for roman type (by circling in black pencil) only in contexts in which they might be mistaken for symbols:

The values of s_{in} and s_{out} both increase with...

clearly should be set as

The values of s_{in} and s_{out} both increase with...

(3) Some latin letters, considered abbreviations of words, are properly roman instead of italic—for example, chemical symbols (O, Ne), most multiletter abbreviations (fcc, ESR, exp, sin), and most units of measure (K, Hz). But the editorial staff of the journal is trained to spot these, and authors need not mark them for roman type unless confusion is especially likely:

$$m_{\text{Ne}}, \text{Res } x$$

The marking of the first example makes it clear that the subscript means "neon," not Ne ($N \times e$). The marking of the second example identifies the function as the residue of x ($\text{Res } x$) instead of the real part of sx ($\text{Re } sx$).

3. Boldface type

(1) Indicate boldface by a single wavy underline in black pencil:

$$\underline{k}, \underline{\nabla}, \underline{A}$$

TABLE V. Handwritten symbols requiring identification. The identification numbers in parentheses in column 2 are taken from Appendix F. "Typeset" versions of latin letters (column 3) are in italic type, the type style appropriate in most mathematical contexts.

Symbol	Identification (No.)	As typeset	Symbol	Identification (No.)	As typeset
\mathcal{A}	cap aye	\mathcal{A}	ψ	lc psi	ψ
\mathbf{a}	lc aye	\mathbf{a}	Ψ	cap psi	Ψ
α	lc alpha	α	ϕ	lc phi	ϕ
\propto	proportional to (R10)	\propto	Φ	cap phi	Φ
∞	infinity (M1)	∞	Σ	cap sigma	Σ
\mathbf{c}	lc cee	\mathbf{c}	Σ	summation (Q29)	Σ
\mathbf{C}	cap cee	\mathbf{C}	\mathcal{S}	lc ess	\mathcal{S}
	vertical bar (B15)		\mathcal{S}	cap ess	\mathcal{S}
/	solidus (Q18)	/	\mathcal{V}	cap vee	\mathcal{V}
'	prime (D15)	'	\mathcal{V}	lc vee	\mathcal{V}
1	one	1	ν	lc nu	ν
ℓ	lc ell	ℓ	γ	lc gamma	γ
\mathbf{e}	lc ee	\mathbf{e}	ν	lc upsilon	ν
\in	an element of (R28)	\in	u	lc you	u
ϵ	lc epsilon	ϵ	μ	lc mu	μ
\mathcal{E}	eh (P7)	\mathcal{E}	\mathcal{W}	cap double-you	\mathcal{W}
\mathcal{K}	cap kay	\mathcal{K}	\mathcal{w}	lc double-you	\mathcal{w}
\mathbf{k}	lc kay	\mathbf{k}	ω	lc omega	ω
κ	lc kappa	κ	x	lc ex	x
π	lc en	π	\mathcal{X}	cap ex	\mathcal{X}
η	lc eta	η	\times	multiplication sign (Q8)	\times
$^\circ$	degrees (M15)	$^\circ$	χ	lc chi	χ
\circ	lc oh	\circ	z	lc zee	z
\bigcirc	cap oh	\bigcirc	Z	cap zee	Z
0	zero	0	2	two	2
ρ	lc rho	ρ	$\langle \rangle$	angular brackets (B19, B20)	$\langle \rangle$
\mathbf{p}	lc pee	\mathbf{p}	\langle, \rangle	less than, greater than (R11, R12)	\langle, \rangle
\mathbf{P}	cap pee	\mathbf{P}	\dagger	dagger (D13)	\dagger
π	lc pi	π	$+$	plus (Q1)	$+$
Π	cap pi	Π			
\prod	product symbol (Q30)	\prod			

will be typeset as

$$\mathbf{k} \cdot \mathbf{p}, \nabla \times \mathbf{A}$$

Note that latin letters marked with a wavy underline will be set boldface roman, to distinguish them further from lightface italic. Dots and multiplication signs between letters marked for boldface will also be set boldface.

(2) Boldface is used for three-vectors, dyadics, some matrices, tensors without indices, etc. It is inappropriate for four-vectors (k), vectors represented by a typical component, such as x_i for $\mathbf{x} = (x_1, x_2, x_3)$, and the magnitude of a vector, H for \mathbf{H} . Subscripts attached to a three-vector should

not be boldface unless they would be so as main characters:

$$\underline{k}_n$$

is correctly marked, to become k_n . When it is essential to distinguish between vectors and tensors, sans serif may be used for tensors.

(3) Some physics journals avoid boldface and represent three-vectors with an overhead arrow: $(\vec{k} \cdot \vec{p})$. Then dyadics may be represented by a double-headed arrow (\overleftrightarrow{k}), and matrices by an underline (\underline{k}). Note that latin letters embellished with arrows will be set lightface roman. Examine a recent issue of the journal to check the details.

4. Subscripts and superscripts

(1) Subscripts and superscripts attached to a symbol are normally set flush against the symbol and should so appear in manuscript:

$$A_1^a, m_{12}^{\prime}, {}_{92}^{238}U$$

If they are to be set staggered, explicitly note the exception in black pencil in the left margin:

$$\overset{\text{set}}{\nearrow} T_{\lambda 0}^{\mu} \overset{\text{set}}{\nwarrow} A^{-1}_{mn}$$

(2) Clarify subscripts and superscripts with a caret if your word processor does not distinguish them, and whenever there is any possibility of mistaking their extent:

$$\text{For } \delta_{\epsilon, \sigma} \text{ we obtain}$$

Draw the caret in black pencil. Do not write

$$\delta_{A, \sigma}$$

which looks as if the carets were to be typeset.

(3) Clarify multilevel indices in black pencil:

$$B_{i, \lambda}, N^{\alpha, \beta}, g^{m, n}$$

Indices of this type are sometimes difficult to typeset and, once typeset, sometimes difficult to read. Use simpler alternatives if possible. For example, if a_n is the n th member of a series, then the n_1 th member has to be a_{n_1} ; but the z component of the vector H_c should be written \underline{H}_{cz} , since c_z has no meaning. Similarly, the square of H^0 should be written $(\underline{H}^0)^2$, not \underline{H}^0 .

More than two levels of indices, as in A^{a_n} , are impractical and unreadable, and should be avoided.

(4) Subscripts and superscripts that precede a symbol should be marked to be spaced off whenever closing up might be misleading:

$$3\gamma_{\frac{1}{2}} F_1(x), 5p^{\frac{1}{2}} S_{1/2}$$

C. DISPLAYED EQUATIONS

Display all numbered and complicated unnumbered equations on separate lines set off from the text above and below. "Complicated" equations are equations hard to accommodate within running text. These include equations longer than about 25 characters,

$$F_0 = N_1 [f(C/(n)) + K(C(n))C(n^2) + \dots], \quad (1)$$

and equations that contain built-up fractions, matrices or matrixlike expressions, multilevel indices, or integral, summation, or product signs with limits:

$$\alpha = \frac{B(E_0) + c}{f_1 + f_2} f(\omega), \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad (2)$$

$$\hat{\beta} = \sum_{i=1}^n P_i$$

1. Multilinear equations

(1) If a displayed equation will not fit on one line, break the equation according to the following models:

$$\langle k + q | V_0 | k \rangle = \langle k + q | V(r, r_0) \sigma(r_0, r) d\tau | k_0 \rangle_0$$

$$+ \langle k + q | v(r - r_0) \sigma(r_0, r) d\tau_0 | k_0 + q \rangle$$

$$\times \langle k + q | 2N_1 | k_0 \rangle + \langle k + q | 2N_2 | k_0 \rangle, \quad (3)$$

$$(\hbar \Gamma_{nk} - \hbar \omega_c) \langle nk | f | n + 1, k \rangle$$

$$= 2le^n E_0 (1 - e^{-\lambda}) (n/2)^{1/2} (n-1)^{1/2} f_{nk}. \quad (4)$$

Operators and signs of relation begin continued lines. When a product is continued from line to line, as in Eq. (3), start the continuation with a multiplication sign.

(2) Align signs of relation in

$$R = (2\pi)^{-N/4} |\chi|^{-1/4} \exp(-\frac{1}{2} \chi_{ij} \hat{q}_i \hat{q}_j)$$

$$= \frac{1}{2} a_{ij} \hat{q}_i \hat{q}_j + b_i \hat{q}_i + c \quad (5)$$

and

$$S(\mathbf{k}, 0) \simeq 1 - \bar{\eta}^2, \quad \mathbf{k} \neq 0$$

$$\simeq 0, \quad \mathbf{k} = 0 \quad (6)$$

or, alternatively,

$$S(\mathbf{k}, 0) \simeq 1 - \bar{\eta}^2 \quad (\mathbf{k} \neq 0)$$

$$\simeq 0 \quad (\mathbf{k} = 0). \quad (7)$$

(3) The journals published by the Institute and its Member Societies are normally printed with two columns per page. Occasionally, when multilinear equations are very long, or when equations contain large matrices or long fraction lines, it is better to print across the page, in a single "wide" column. Authors should try to avoid the need for this, however, since it always makes the paper harder to read. If possible, define the component parts of such long equations in terms of appropriate symbols. In any case, the decision to set in a one- or two-column width should be left to the editorial office.

2. Equation numbers

(1) Only displayed equations may be numbered.

(2) The preferred style is to number equations consecutively throughout the text with arabic numerals in parentheses: (1), (2), (3), etc. Numbering by section is also acceptable, if the section number is made part of the equation number: (2.1), (2.2), (2.3), etc. In appendixes use the numbering sequence (A1), (A2), (A3), etc.

(3) Place equation numbers flush with the right margin. Leave a space at least two characters wide between an equation and its number.

(4) An equation number should be centered beside a group of equations identified by one number, as in Eqs. (2), (6), and (7) on this page. It should be aligned with the last line of a multilinear equation, as in Eqs. (3)–(5).

3. Built-up fractions, slashed fractions, and negative exponents

(1) A fraction can be represented in three alternative ways: "built up" with a fraction line,

$$\frac{x + y}{z},$$

or "slashed" with a solidus,

$$(x + y)/z,$$

or with negative exponents,

$$(x + y)z^{-1}.$$

In running text one of the last two forms must invariably be the choice, since built-up fractions will not fit. In displayed equations, use the slashed form or negative exponents for short, simple fractions, but use the built-up form for fractions that are long and complex. That is, write

$$pq^2/r + (n/y)^2 = 0$$

or, better,

$$(pq^2/r) + (n/y)^2 = 0$$

or, better still,

$$pq^2r^{-1} + n^2y^{-2} = 0$$

but

$$\alpha = \frac{B(E_0) + c}{f_1 + f_2} f(\omega)$$

is easier to read than

$$\alpha = [B(E_0) + c](f_1 + f_2)^{-1} f(\omega)$$

or

$$\alpha = \{[B(E_0) + c]/(f_1 + f_2)\} f(\omega).$$

(2) Do not mix built-up and slashed forms unnecessarily within one equation. Write

$$\frac{a}{b} = \frac{B(E_0) + c}{f_1 + f_2} f(\omega)$$

instead of

$$a/b = \frac{B(E_0) + c}{f_1 + f_2} f(\omega).$$

On the other hand, a built-up fraction within the numerator or denominator of another built-up fraction is ungainly and hard to set. Write

$$B' = \frac{[3J/(J+1)]T_N m^2}{En - Bn^2 b^2 n^2 T_N}$$

and, similarly,

$$A = \frac{x}{y} \begin{pmatrix} (a+b)/n & 0 \\ 0 & (a+c)/n \end{pmatrix}.$$

Use of a negative exponent is often the best alternative when slashing fractions within built-up fractions would be very awkward:

$$B = \frac{[3J/(J+1)]T_N m^2}{En - Bn^2 - n^2 m^2 T_N} \left(\frac{B(E_0) + c}{f_1 + f_2} f(\omega) \right)^{-1}.$$

(3) As a rule, use built-up fractions in displayed equations containing integral, summation, or product signs. In displayed equations these signs are usually large, and therefore no particular economy results from using the solidus. Write

$$\frac{a}{b} = \int_0^\pi dx \quad \text{and} \quad \frac{a}{b} = \sum_{i=1}^\infty a_i$$

instead of

$$a/b = \int_0^\pi dx \quad \text{and} \quad a/b = \sum_{i=1}^\infty a_i.$$

(4) Always use slashed fractions in subscripts and superscripts:

$$\int_z^{\pi/2} \left(\frac{t-z}{t-\alpha/(1-\alpha)} \right)^{3/2} dt.$$

(5) Any rational fraction can be put on one line in the special upright form, for example, $\frac{1}{3}$. This form is awkward in subscripts and superscripts [see point (4) above]; elsewhere, use the upright form instead of forms with a solidus as often as possible. Write $\frac{1}{3}x$ instead of $x/3$, $(1/3)x$, or $1/3x$; never write $1/3x$ unless you mean $1/(3x)$. Write

$$\sin(\frac{1}{3}x) \quad \text{and} \quad \frac{1}{3} \sin x$$

instead of

$$\sin(x/3) \quad \text{and} \quad (\sin x)/3;$$

never write $\sin x/3$.

(6) When using the solidus, make sure that your meaning is unambiguous:

$$a/b + c \quad \text{means} \quad (a/b) + c$$

but use the parentheses, or write $ab^{-1} + c$, to allay any doubts. Never write

$$a/b/c;$$

write instead

$$(a/b)/c \quad \text{or} \quad a/(b/c)$$

(7) Make sure that the fraction line clearly delimits the numerator and denominator of a built-up fraction. Does

$$\ln \frac{a}{b} \quad \text{mean} \quad \frac{\ln a}{b} \quad \text{or} \quad \ln\left(\frac{a}{b}\right)?$$

Do not leave the resolution of such ambiguities to copyeditors.

4. Integral, summation, and product signs

In running text, integral, summation, and product signs cannot be full sized, and symbols attached to them must always be subscripts and superscripts: \int_c , $\sum_{i=1}$, \prod_a . In displayed equations, integral, summation, and product signs can be full sized, and there is ample room for symbols above or below summation and product signs:

$$\int_c \quad \sum_{i=1} \quad \prod_a$$

D. RECOMMENDED NOTATION

1. Parentheses

(1) According to accepted convention, one works outward with parentheses according to the scheme

$$\{[()]\}$$

Boldface parentheses are available; once the above scheme is exhausted, repeat the sequence in boldface. Also use boldface parentheses to enclose the argument of a function if the argument itself contains parentheses.

(2) Small parentheses and brackets are available for use in subscripts and superscripts. Oversize parentheses and brackets are available for use in displayed equations around expressions containing built-up fractions or integral, summation, or product signs.

(3) In general, too many parentheses are better than too few. But wholesale use of parentheses obscures rather than

clarifies, and—especially in displayed equations—wastes labor and space. All of the parentheses in the following example are extraneous:

$$\left[\left(\frac{(l+2)}{a^2} \right) + \left(\frac{2x^2}{(0.01)\pi/a} \right) \right] = 0.$$

(4) The introduction of special brackets that do not simply group, but also define, what they enclose does not affect the usual scheme:

$$[|\hbar(\langle E \rangle^{1/2} + m)|^2 - \Delta S]^{-1}.$$

But if any of the common parentheses or brackets are used with a special meaning, they should not then be used for grouping:

$$\frac{1}{2}\{ -i(L_1[L_2, L_3] + K_j) \}$$

In such cases, it is wise to identify the special brackets when they first appear to assure copyeditors that the unusual ordering is no slip.

(5) The following special uses of parentheses and brackets are recommended:

plane, or set of parallel planes	(111)
point designated by coordinates	(x, y, z)
vector written in components	(H_x, H_y, H_z)
function of a function	$F(f(x))$
direction	[111]
class (group) of symmetry-equivalent planes	{111}
class (group) of symmetry-equivalent directions	\langle 111 \rangle

2. Mathematical functions set in roman type

(1) Standard abbreviations for functions set in roman type are given in Appendix D. Apart from these standard functions, the use of multiletter symbols for single quantities (such as “TKE” for total kinetic energy, or “BR” for branching ratio) is improper.

There is usually no need to mark the standard functions for roman type [see point (3) of Sec. IV B 2]; but take care, when using such functions, to respect the following conventions.

(a) A function is closed up with its argument:

$$\text{Tr}Q, \tan(\eta^2/\mu)$$

(b) The function of a product may be written without parentheses:

$$\sin xa \text{ means } \sin(xa).$$

But put parentheses around the product whenever there is a chance of confusion:

$$\sin(x^2a^{3/2}), \sin(\frac{1}{2}x).$$

Note, however, that an argument is assumed to stop as soon as another function appears:

$$\sin x \cos a \text{ means } (\sin x) \cos a.$$

(c) $\sin x/a$ is ambiguous. Write $(\sin x)/a$ or $\sin(x/a)$, depending on the intended meaning.

(d) An argument stops at a plus or minus sign:

$$\sin x + a \text{ means } (\sin x) + a$$

(e) A plus or minus sign should never directly follow a function. Write

$$\sin[-(x+a)] \text{ rather than } \sin-(x+a)$$

(2) The decision to use \exp or its italic alternative e depends on the nature of the argument. Use e if the argument is simple enough to make a readable superscript:

$$e^{B/T}, e^{-x^2}$$

If the argument is long, or contains complicated indices, a built-up fraction, or an integral, summation, or product sign, use \exp instead:

$$\exp[-h(t) + (\omega - \epsilon)t], \exp(x_{vw}^2 a^{3/2}),$$

$$\exp\left(\frac{a+bx}{c+dy}\right), \exp\left(-\sum_{R,S} h_{RS}(0)\right).$$

Be consistent. Try not to use \exp and e in proximity or for the same argument.

3. Radicals and bars over groups of symbols

The “roof” of a radical or a bar can be set over simple expressions, as in \sqrt{a} , $\sqrt{a+b}$, and \overline{AB} , but may be awkward over complicated ones. An exponent 1/2 or special brackets may then be used instead: $(a^2 + b^2)^{1/2}$, $\langle A^n B \rangle_{av}$. A bare, “unroofed” radical may also be used, but always with a liberal use of parentheses to avoid ambiguity. Write:

$$\begin{aligned} \sqrt{(x^3a)} & \quad \text{or} \quad (\sqrt{x^3})a, \\ \sqrt{(x^3/a)} & \quad \text{or} \quad (\sqrt{x^3})/a, \\ \sqrt{(x^3+a)} & \quad \text{or} \quad (\sqrt{x^3})+a, \end{aligned}$$

depending on your meaning. Clearly, as substitutes for roofed radicals, forms with an exponent 1/2 are usually less troublesome and more readable than forms with a bare radical.

4. Multiplication signs

(1) Use a multiplication sign for the vector product of three-vectors:

$$\nabla \times \mathbf{A} \quad (\text{not } \nabla \wedge \mathbf{A}).$$

As a rule, do not use multiplication signs for simple products. Write

$$2ab \text{ instead of } 2 \times ab \text{ or } 2a \times b$$

Exception is made for the continuation of a product from line to line [see point (1) of Sec. IV C 1], and for the following special cases:

$$3.4 \times 10^{-6} \text{ eV}, 3 \times 2 \times 1 \text{ mm}$$

$$1 \times 3 \times 5 \times \cdots \times (2n-1), 2f \times \frac{1}{2}g.$$

(2) Centered dots may be used in compound units: $4 \text{ g} \cdot \text{cm}^2 \cdot \text{s}^{-2}$. Their principal use is in inner products of vectors, dyadics, and the like:

$$\mathbf{k} \cdot \mathbf{p}, \mathbf{l} \cdot \mathbf{g} \cdot \mathbf{s}.$$

Do not use dots for simple products, or for any other kind of multiplication. Write

$$2ab \text{ instead of } 2 \cdot ab \text{ or } 2a \cdot b.$$

5. Mathematical terms

The American Institute of Physics strongly recommends the usage of the following symbols:

approximately equal to	\approx or \simeq
proportional to	\propto
tends to	\rightarrow
asymptotically equal to; of the order of magnitude of	\sim
of the order of (in the mathematical sense)	$O(\)$

complex conjugate of A	A^* (not \bar{A})
Hermitian conjugate of matrix A	A^\dagger
transpose of matrix A	A^{\sim}, A^T
unit vector \mathbf{k}/k	$\hat{\mathbf{k}}$

Customary usage in physics (in contrast to mathematics) is to denote complex conjugation by an asterisk rather than an overbar, and Hermitian conjugation by a superscript dagger (not a plus sign!) rather than an asterisk. Transposition should be denoted by a superscript tilde, or, better, a roman T, not an overtilde.